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# MODELING THE MAXIMUM DAILY RAINFALL IN THE MUNICIPALITY OF JOÃO PESSOA-PB, BRAZIL, USING THE EXTREME VALUE THEORY 

# Modelando a chuva máxima diária no município de João PessoaPB por meio da Teoria dos Valores Extremos 

## Modelización de la iluvia máxima diaria en la municipio de João Pessoa-PB utilizando la Teoría de Valores Extremos

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#### Abstract

Heavy rains have caused numerous disturbances in several Brazilian regions, particularly in the Northeast. This study aimed to model the maximum daily rainfall in the municipality of João Pessoa-PB, Brazil. The historical series data from 1980 to 2019 were analyzed using extreme value theory (EVT), with Gumbel and generalized extreme value (GEV) distribution fits. The results showed that the Gumbel distribution had the best fit to the data from January to August and October, with parameters estimated by the maximum likelihood method. The GEV distribution was the most suitable for September, November, and December, which typically have lower rainfall levels. Moreover, return level estimates point to the occurrence of heavy rain s due to rainfall intensity in a single day for return periods of 2,5 , and 10 years. These results can provide subsidies for planning public policies to reduce heavy rain impacts.


Keywords: Environmental disaster. Gumbel. Rainfall. Flooding.

Resumo: As chuvas extremas têm causado inúmeros impactos em várias regiões brasileiras, principalmente no Nordeste. Este estudo teve como objetivo modelar a chuva máxima diária no município de João PessoaPB, Brasil. Os dados da série histórica de 1980 a 2019 foram analisados usando a teoria dos valores extremos (EVT), com ajustes da Gumbel e da distribuição Generalizada de Valores Extremos (GEV). Os resultados mostraram que a distribuição de Gumbel se ajustou melhor aos dados de janeiro a agosto e outubro, com parâmetros estimados pelo método de máxima verossimilhança. A distribuição GEV foi a mais adequada para setembro, novembro e dezembro, que normalmente apresentam níveis de chuvas mais baixos. Além disso, as estimativas do nível de retorno apontam para a ocorrência de fortes chuvas devido à sua intensidade em um único dia para períodos de retorno de 2,5 e 10 anos. Esses resultados podem fornecer subsídios para o planejamento de políticas públicas à redução dos impactos das chuvas extremas.
Palavras-chave: Desastre Ambiental. Gumbel. Chuva. Inundação.

Resumen: Las lluvias extremas han causado innumerables impactos en varias regiones brasileñas, principalmente en el Nordeste. Este estudio tuvo como objetivo modelar la lluvia máxima diaria en la ciudad de João Pessoa-PB, Brasil. Los datos de la serie temporal de 1980 a 2019 se analizaron utilizando la teoría del valor extremo (EVT), con ajustes de Gumbel y del valor extremo generalizado (GEV). Los resultados mostraron que la distribución de Gumbel se ajusta mejor a los datos de enero a agosto y octubre, con parámetros estimados por el método de máxima verosimilitud. La distribución de GEV fue la más adecuada para los meses de septiembre, noviembre y diciembre, que normalmente tienen menores niveles de iluvias. Además, las estimaciones del nivel de retorno apuntan a la ocurrencia de eventos extremos debido a la intensidad de las lluvias en un solo día para períodos de retorno de 2,5 y 10 años. Estos resultados pueden proporcionar subsidios para la planificación de políticas públicas para reducir los impactos de las lluvias extremas.

Palabras clave: Desastre ambiental. Gumbel. Iluvias. Inundación.

## 1. INTRODUCTION

Extreme events related to weather variables, such as rainfall, temperature, and humidity, have occurred more frequently in recent years, causing countless problems for society, many of which are catastrophic in nature, negatively impacting the quality of life of populations (ZANELLA; SALES; ABREU, 2009). Extreme weather events can cause heavy rains, leading to socio-environmental problems such as reduced crop yields, in addition to the occurrence of floods, overflows, and torrents, which, in turn, cause several inconveniences to the locals (ANDRADE; PINHEIRO; DOLIF NETO, 2015; OLIVEIRA; LIMA, 2019).

Damages and inconveniences caused by extreme weather events have aroused the need for and importance of more research on this subject. Regarding rainfall, socioenvironmental planning can be improved by including detailed statistical analysis, avoiding disasters by heavy rains through probability calculations and one-day maximum rainfall estimates for different return periods (COTTA; CORREA; ALBUQUERQUE, 2016; MEDEIROS; ALVES; SOUZA, 2019; WANDERLEY et al., 2018).

Extreme value theory (EVT) occupies a fundamental role in the statistical modeling of extreme rains, which are associated with rare low-probability events (FERREIRA; LISKA, 2019). The EVT has been widely used in climatology and hydrology fields, with Gumbel and Generalized Extreme Value (GEV) distributions fits, to obtain maximum rainfall return level estimates (DE PAOLA et al., 2018; GONZÁLEZ-ÁLVAREZ et al., 2019; OSEl et al., 2021). De Paola et al. (2018) used the GEV distribution to model extreme rainfall in the African cities of Dar Es Salaam (Tanzania) and Addis Ababa (Ethiopia). González-Álvarez et al. (2019) performed a mapping of the maximum daily rainfall in several rainfall stations located in the Colombian Caribbean Region, concluding that the GEV and Gumbel distributions provided the best fits in $47.2 \%$ and $34.3 \%$ of the pluviometric stations, respectively. Osei et al. (2021) estimated return levels, by adjusting the Gumbel distribution, for a period of 5 to 100 years of maximum annual rainfall in the Pra River Basin, Ghana, West Africa.

In Brazil, several studies have modeled the maximum rainfall through the theory of extreme values. Cotta, Correa, and Albuquerque (2016) modeled the maximum monthly rainfall data in Vitória's municipality, Espírito Santo, through EVT and found that the Gumbel distribution was suitable to fit the data under study. Santos et al. (2018) investigated the monthly extremes of rainfall in Cacoal, Rondônia State, and identified that the Gumbel
distribution was one of the most suitable for obtaining return levels estimates. Back and Cadorin (2020) estimated extreme annual rainfall in Amapá state, Brazil, using EVT and observed that GEV and Gumbel distributions were suitable to obtain annual maximum rainfall estimates with better results for GEV.

The capital of Paraíba State has been negatively impacted by natural disasters in recent decades caused by heavy rains, leading to falling trees, landslides, flooding in residential areas, and aggravation of social and sanitary problems of residents living in areas with higher socioenvironmental vulnerability (PEREIRA et al., 2012). It is noteworthy that studies like ours provide useful information to be made available for public policy managers. In this sense, the Brazilian Atlas of Natural Disasters (CEPED, 2013) shows 136 occurrences of floods between 1992 and 2014, resulting in disasters in the entire state of Paraíba, with João Pessoa being the most affected municipality. Given the importance of information on extreme rainfall returns, out study aimed to model the maximum daily rainfall using EVT in the municipality of João Pessoa, Paraíba state, Brazil. We believe our results can assist in planning public policies to minimize impacts from heavy rains in the region.

## 2. METHODOLOGY

### 2.1. Material

The Northeast of Brazil has a large territorial area, comprising nine federative units. This region has different climates, including tropical, semiarid, and humid equatorial (OLIVEIRA; LIMA, 2019). The tropical climate has two well-defined seasons, one dry and the other rainy, and is present throughout the coastal region, especially on the coast of Paraíba State, with the coastal municipality of João Pessoa as its capital. The atmospheric systems of the municipality of João Pessoa act mainly in low-latitude equatorial areas, usually promoting atmospheric stability in late winter and spring, but causing instability in the seasonal period of summer, fall, and early winter (MENDONÇA; DANNI-OLIVEIRA, 2007; ROCHA FILHO et al., 2019). The intense rainfall in João Pessoa can be explained by the influence of the phenomena of the Intertropical Convergence Zone, Upper Air Cyclonic Vortices, Instability Lines and the Atlantic Polar Mass (PEREIRA et al., 2012).

This study was performed using daily rainfall data from João Pessoa municipality,
capital of Paraíba state, in northeastern Brazil (Figure 1). The dataset refers to the period from January 1980 to December 2019, totaling 14,611 daily observations.

The data were obtained from a conventional gauge station located in João Pessoa ( $7^{\circ} 05^{\prime} 43.0^{\prime \prime} \mathrm{S}, 34^{\circ} 50^{\prime} 55.0^{\prime \prime} \mathrm{W}$, and 9.67-meter altitude; Figure 1), which belong to the Brazilian National Institute of Meteorology (INMET - http://portal.inmet.gov.br/). To build the data series, the maximium daily rainfall was extracted for each month as a monthly maximum. Altogether, 480 observations were obtained, 40 for each monthly series.

Figure 1 - Geographic situation of the conventional rainfall station in the municipality of João Pessoa, Paraíba.


Source: Elaborated by the authors (2021).

### 2.2. Methods

The descriptive measures minimum, maximum, mean, median, standard deviation, and coefficient of variation (CV) were used for exploratory analysis. Graphical analyses were performed using Boxplot.

The Wald-Wolfowitz test was used to detect stationarity in each maximum monthly rainfall series. The test statistic is non-parametric and is used to check whether a time series has biased observations, considering the null hypothesis $\left(H_{0}\right)$ that the series is stationary against the alternative hypothesis ( $H_{1}$ ) of non-stationarity (WALD; WOLFOWITZ, 1940). The test consists of counting the number of values above and below the median in a data series. This number of oscillations is called run and is also tested for normal distribution. The sampling distribution of the total number of runs can be approximated by a normal distribution, with the mean and variance given by the respective equations:

$$
\begin{equation*}
E(u)=2 n_{1} n_{2} /\left(n_{1}+n_{2}\right) \text { and } \operatorname{Var}(u)=2 n_{1} n_{2}\left[2 n_{1} n_{2}-\left(n_{1}+n_{2}\right)\right] /\left[\left(n_{1}+n_{2}\right)^{2}\left(n_{1}+n_{2}-1\right)\right] . \tag{1}
\end{equation*}
$$

In Eq. (1) the term $u$ is the number of runs, $n_{1}$ is the number of observations higher than or equal to the median, and $n_{2}$ is the number of observations lower than the median. Thus, using an asymptotic approximation, the test statistic is given as follows:

$$
\begin{equation*}
Z=[u-E(u)] / d p(u) . \tag{2}
\end{equation*}
$$

In Eq. (2) the random variable $Z$ follows a standard normal distribution and the terms $E(u)$ and $d p(u)$ represent the expected values and the standard deviation of the variable $u$, respectively.

The Ljung-Box (L-B) test was used to check the independence between observations throughout each series. Therefore, the L-B test assesses the hypothesis of independence of a time series (LJUNG; BOX, 1978). This test is essential since the adopted estimation method (maximum likelihood) assumes that the observations are independent.

The generalized extreme value (GEV) distribution is composed of a family of distributions, namely Gumbel (type I), Fréchet (type II), and Weibull (type III), which are used in extreme value distributions and applied in different science fields (NAGHETTINI; PINTO, 2007). The GEV distribution has three parameters, namely: $\xi$ (shape), $\sigma$ (scale), and $\mu$ (position), among which $\xi$ determines which distribution is best fitted to the data. A Gumbel distribution is given when $\xi$ is statistically equal to zero in the GEV distribution. The Fréchet distribution, in turn, is given when $\xi>0$, while the Weibull distribution is given when $\xi<0$.

The GEV distribution was fitted and then compared with the Gumbel distribution. The cumulative probability function of the GEV distribution (Equation 3) is given as follows:

$$
\begin{equation*}
F(x)=\exp \left\{-[1+\xi((x-\mu) / \sigma)]^{-1 / \varepsilon}\right\} . \tag{3}
\end{equation*}
$$

Position and scale parameters can be found in the Gumbel distribution (Equation 4), and its accumulated distribution function is given as follows:

$$
\begin{equation*}
F(x)=\exp \{-\exp [-((x-\mu) / \sigma)]\} . \tag{4}
\end{equation*}
$$

After checking the stationarity and independence of observations in the series, the next step consisted of fitting probability distributions, obtaining parameter estimates by the maximum likelihood method. The estimators of the parameters presented in Equations (3) and (4) are obtained through functions that depend on the observed data, however, these functions require the use of numerical methods to determine the solution. Maximum likelihood estimations of these parameters can be consulted in the works by Boudrissa et al.
(2017) and Vivekanandan (2017). Then, the likelihood ratio test was used to compare the GEV and Gumbel distributions, as follows:

$$
\begin{equation*}
T L R=-2\left[/\left(\theta_{G}\right)-l\left(\theta_{G E V}\right)\right] . \tag{5}
\end{equation*}
$$

In Equation (5), $/\left(\theta_{G}\right)$ and $/\left(\theta_{G E V}\right)$ represent the logarithms of the maximum likelihood of the Gumbel and GEV functions, respectively (MEDEIROS; ALVES; SOUZA, 2019). This test checks whether $\xi$ is statistically equal to zero by adopting the null hypothesis $\left(H_{0}\right) \xi=0$. The Gumbel distribution is chosen when $\xi$ is statistically equal to zero. The $p$-value of this test is obtained by the chi-square distribution with one degree of freedom.

The Kolmogorov-Smirnov ( $\mathrm{K}-\mathrm{S}$ ) test was adopted as an adherence test, after selecting the probability distribution to verify whether the chosen distribution is suitable for the data. The K-S is a non-parametric test, and its statistic is constituted by the discrepancy between observed $\mathrm{F}(\mathrm{x})$ and theoretical $\mathrm{G}(\mathrm{x})$ distribution values and is obtained as follows:
$D=\operatorname{supremum}[F(X)-G(X)]$.
In Equation (6), $D$ value is then compared with at tabulated value and approximated to $1.63 / \vee n$ when a $1 \%$ significance level is adopted. Therefore, the distribution is suitable for the data if the $D$ is lower than the tabulated value. Additionally, in all hypothesis tests in this research, a $1 \%$ significance level was adopted.

Finally, after selecting the probability distribution function, the last step was to obtain return levels for the distribution in question, using parameter estimates. According to Naghettini and Pinto (2007), the return time (RT) is the average time interval (in years) that separates the occurrence of an event of a known dimension from another with an equal or higher dimension. The equation that represents $R T$ is given as follows:

$$
\begin{equation*}
R T=1 /[1-F(x)] \tag{7}
\end{equation*}
$$

In Equation (7), RT represents the return time and $F(x)$ is the GEV or Gumbel accumulated distribution function. After fitting the GEV and Gumbel distributions, return level estimates were obtained as in Equations (8) and (9), respectively:

$$
\begin{equation*}
\text { GEV: } x_{p}{ }^{*}=\mu^{*}-\left(\sigma^{*} / \zeta^{*}\right)\left\{1-[-\ln (1-p)]^{-\xi^{*}}\right\}, \xi \neq 0 \tag{8}
\end{equation*}
$$

(9) Gumbel: $x_{p}^{*}=\mu^{*}-\sigma^{*}\{\ln [-\ln (1-p)]\}, \xi=0$

In Equations (8) and (9), $x_{p}{ }^{*}$ is used to estimate return levels associated with $R T$ in $1 / p$ years. Estimates of $\mu^{*}, \sigma^{*}$ and $\xi^{*}$ estimators were obtained by the maximum likelihood method. Moreover, asymptotic intervals were determined at the $99 \%$ confidence level for
return levels, considering periods of 2,5 , and 10 years. To exemplify a use of Equation (9), suppose that the estimates of $\mu^{*}$ and $\sigma^{*}$ were equal to 40 and 20 , respectively. For a return time of 5 years $(R T=5)$, we have that the value of $p=1 / R T$ would be 0.20 . The estimate of the level return would be given by: $\mu^{*}-\sigma^{*}\{\ln [-\ln (1-p)]\}=40-20\{\ln [-\ln (1-0,20)]\}=70$. Thus, for an interval of 5 years, the maximum daily rainfall is expected to be equal or higher than 70 mm .

All statistical analyses were performed using the $R$ software (R CORE TEAM, 2018), with support from the libraries ggplot2 (WICKHAM, 2009), randtests (CAEIRO; MATEUS, 2014), extRemes (GILLELAND; KATZ, 2016), and evd (STEPHENSON, 2002).

## 3. RESULTS AND DISCUSSIONS

In Figure 2, there is the climogram of the average monthly rainfall and temperature in the municipality of João Pessoa, based on the period 1980 to 2019. It can be seen in an inversely proportional relationship between these variables, months with high temperatures have low rainfall rates. This relationship can be seen in the months of June and July, which have the lowest temperatures and highest average rainfall in this region.

Figure 2 - Climogram representing monthly average rainfall (lightblue bars, left $Y$ axis) and temperature (red line, right Y axis) based on the 40 -year period (1980-2019) data obtained from the municipality of João Pessoa, Paraíba state, Brazil.


Source: Elaborated by the authors (2021).

For a better understanding of the behavior of maximum rainfall in João Pessoa, it is
necessary an exploratory analysis through descriptive measures (Table 1) and the Box Plot graph (Figure 3). Table 1 shows a high variation in maximum daily rainfall over the year, reaching 194.00 mm in June and 39.2 mm in November. Fluctuations within each month were also expressive, ranging from 15.6 to 194 mm in June and 3.2 to 159.2 mm in September. However, less rainy months, such as November and December, showed, in general, higher variability, with CV values of 103.5 and $120.7 \%$, respectively, that is, the highest values among all months. On the other hand, the lowest CV value was observed in July (55.6\%). The highest maximum means were recorded from March to July, with values above 50 mm in a single day.

Table 1 - Statistical summary of maximum daily rainfall from 1980 to 2019 in the municipality of João Pessoa, PB, Brazil.

| Month | Minimum | Maximum | Mean | Median | Standard <br> deviation | CV (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| January | 0.0 | 118.2 | 31.2 | 23.1 | 27.5 | 88.1 |
| February | 2.9 | 119.8 | 42.5 | 35.4 | 32.8 | 76.9 |
| March | 0.0 | 152.6 | 55.7 | 51.5 | 35.9 | 64.4 |
| April | 8.5 | 149.0 | 59.9 | 53.5 | 34.8 | 58.0 |
| May | 9.6 | 186.0 | 68.7 | 60.8 | 41.8 | 60.7 |
| June | 15.6 | 194.0 | 78.0 | 61.6 | 46.0 | 59.1 |
| July | 5.6 | 147.2 | 69.5 | 63.7 | 38.7 | 55.6 |
| August | 0.0 | 116.0 | 38.5 | 28.1 | 30.6 | 79.3 |
| September | 3.2 | 159.2 | 28.0 | 17.9 | 27.7 | 98.5 |
| October | 0.3 | 41.0 | 11.4 | 10.5 | 8.5 | 73.7 |
| November | 0.0 | 39.2 | 9.3 | 6.9 | 9.7 | 103.5 |
| December | 0.0 | 87.4 | 14.3 | 9.9 | 17.3 | 120.7 |

Source: Elaborated by the authors (2021).

Pereira et al. (2016) also analyzed a historical series of rainfall from João Pessoa (1981 to 2016) and observed results similar to ours. These authors also reported heavy rains from April to July (above 100 mm in some cases), with most of them registered in June (13 occurrences) and July (8 occurrences).

Figure 3 shows that the study region has two distinct periods regarding maximum daily rainfall distribution, with median values below 50 mm from August to February and maximum median values above 50 mm from March to July. The cause for these events between March
and May is the influence of the intertropical convergence zone (ITCZ), which intensifies in March, as well as the upper air cyclonic vortex (UACV) and instability lines (IL), which act frequently between March and May. Heavy rains in June and July is influenced by easterly waves (EW) and the Atlantic polar mass (APM), as all these systems cause rain and can occur alone or in association (PEREIRA et al., 2012; ROCHA FILHO et al., 2019).

Figure 3 - Boxplot for the maximum daily rainfall in the municipality of João Pessoa, Paraíba state, Brazil, from 1980 to 2019.


Source: Elaborated by the authors (2021).

Table 2 shows the results of stationarity, independence, and likelihood ratio tests and parameter estimates that define the GEV shape. In all months, the stationarity hypothesis was not rejected ( $\mathrm{p}>0.01$ ). Likewise, the null hypothesis that the series had independent observations was not rejected since every month had a $p$-value above the $1 \%$ significance level. Therefore, the GEV and Gumbel distributions can be estimated. Between January and August, and in October, the hypothesis that the shape parameter is statistically equal to zero was not rejected and, therefore, the Gumbel distribution was suitable for these months. However, the Gumbel distribution was not suitable for September, November, and December, with GEV being considered the best option.

Table 2 - P-values of the Wald-Wolfowitz (pWW), Ljung-Box (pLB), and likelihood ratio (pLRT) tests, and shape parameter estimates of the GEV distribution followed by the lower (LL) and upper limits (UL) at a $99 \%$ confidence interval for the maximum daily rainfall in the municipality of João Pessoa,

Paraíba state, Brazil.

| Month | pWW | pLB | Shape parameter (GEV) |  |  |  |  |  |  | LRT | pLRT |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | LL | Estimate | UL |  |  |  |  |  |  |
| January | 0.5271 | 0.5536 | -0.0736 | 0.3350 | 0.7436 | 5.9816 | 0.0146 |  |  |  |  |
| February | 0.5217 | 0.2747 | -0.2920 | 0.2061 | 0.7043 | 1.3252 | 0.2498 |  |  |  |  |
| March | 0.5217 | 0.0762 | -0.3901 | -0.0380 | 0.3140 | 0.0715 | 0.7891 |  |  |  |  |
| April | 0.7487 | 0.5708 | -0.3516 | -0.0156 | 0.3204 | 0.0134 | 0.9076 |  |  |  |  |
| May | 0.0249 | 0.1850 | -0.2476 | 0.0884 | 0.4244 | 0.5033 | 0.4780 |  |  |  |  |
| June | 0.5217 | 0.6410 | -0.2114 | 0.1789 | 0.5693 | 1.4668 | 0.2259 |  |  |  |  |
| July | 0.5217 | 0.7392 | -0.6419 | -0.1309 | 0.3801 | 0.3884 | 0.5333 |  |  |  |  |
| August | 0.5217 | 0.7951 | -0.0714 | 0.2844 | 0.6403 | 4.7972 | 0.0285 |  |  |  |  |
| September | 0.3365 | 0.4937 | 0.0764 | 0.4542 | 0.8320 | 14.69 | $0.0001^{* *}$ |  |  |  |  |
| October | 0.7487 | 0.3161 | -0.2882 | 0.1147 | 0.5175 | 0.6559 | 0.4180 |  |  |  |  |
| November | 0.3365 | 0.1509 | -0.0101 | 0.4466 | 0.9034 | 10.453 | $0.0012^{* *}$ |  |  |  |  |
| December | 1.0000 | 0.5473 | 0.0549 | 0.5679 | 1.0810 | 16.454 | $0.0000^{* *}$ |  |  |  |  |

The symbol "**" indicates rejection of the null hypothesis of each test at the $1 \%$ level of significance.
Source: Elaborated by the authors (2021).

In agreement with our results, the study by Hartmann et al. (2011) observed that the Gumbel distribution had a good fit for the maximum monthly rainfall within the Presidente Prudente, São Paulo state, Brazil. They also reported that rainfall estimates obtained by the maximum likelihood method were consistent and could accurately reproduce the rainfall regime in the region.

We used the Kolmogorov-Smirnov adherence test to verify the suitability of distributions (Gumbel or GEV) fitted to the data in each month. Table 3 shows the best-fitted distribution and parameter estimates. According to the K-S test, the D statistic value was lower than 0.26 ; therefore, the models proposed for the months were suitable for the rainfall data sample at a $1 \%$ significance level.

Table 3 - Parameter estimates of the Gumbel and GEV distributions and K-S test result for suitability of the distribution fitted to the maximum rainfall data series.

| Month | Distribution | Parameter |  |  |  | Statistic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\boldsymbol{\mu}^{*}$ | $\boldsymbol{\sigma}^{*}$ | $\boldsymbol{\xi}^{*}$ |  |  |
| January | Gumbel | 19.7788 | 17.4617 | - | 0.1091 | 0.7272 |
| February | Gumbel | 27.8940 | 23.9910 | - | 0.0873 | 0.9207 |
| March | Gumbel | 39.1592 | 28.7474 | - | 0.1054 | 0.7653 |
| April | Gumbel | 44.0397 | 27.3430 | - | 0.0708 | 0.9880 |
| May | Gumbel | 50.4742 | 30.3263 | - | 0.0964 | 0.8510 |
| June | Gumbel | 57.5727 | 33.1442 | - | 0.1356 | 0.4164 |
| July | Gumbel | 51.1746 | 31.9447 | - | 0.1225 | 0.5442 |
| August | Gumbel | 25.5401 | 20.0143 | - | 0.1250 | 0.5592 |
| September | GEV | 15.2322 | 10.2531 | 0.4542 | 0.0993 | 0.8247 |
| October | Gumbel | 7.7763 | 6.1407 | - | 0.1080 | 0.7383 |
| November | GEV | 4.3715 | 4.2089 | 0.4466 | 0.0834 | 0.9432 |
| December | GEV | 5.8646 | 5.9198 | 0.5680 | 0.1035 | 0.7844 |

Source: Elaborated by the authors (2021).

Our results comply with Santos et al. (2014), who also used the K-S test to verify sample adherence at a $1 \%$ probability level. These authors analyzed the annual maximum daily rainfall in the municipality of Mossoró, Ceará state, Brazil, in the from 1964 to 2011, and reported a good fit for the Gumbel probabilistic model.

Table 4 shows the monthly return level estimates and respective confidence intervals for 2,5 , and 10 years of return. The highest return levels for a maximum daily rainfall were recorded in June, with a maximum one-day rainfall estimate for a two-year interval expected to exceed 69.72 mm , ranging from 57.64 to 81.80 mm at a $99 \%$ confidence level.

Table 4 - Return level estimates $\left(x_{p}{ }^{*}\right)$ followed by the lower (LL) and upper (UL) limits at 99\% confidence intervals for the maximum daily rainfall (mm) in the municipality of João Pessoa, Paraíba state, Brazil.

| Month | 2 years |  |  | 5 years |  |  | 10 years |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | LL | $x_{p}{ }^{*}$ | UL | LL | $x_{p}{ }^{*}$ | UL | LL | $x_{p}{ }^{*}$ | UL |
| January | 19.85 | 26.18 | 32.50 | 35.91 | 45.97 | 56.03 | 45.98 | 59.07 | 72.16 |
| February | 27.93 | 36.68 | 45.44 | 50.12 | 63.87 | 77.63 | 64.08 | 81.88 | 99.67 |
| March | 39.19 | 49.69 | 60.19 | 66.10 | 82.27 | 98.45 | 83.07 | 103.85 | 124.62 |
| April | 44.09 | 54.06 | 64.03 | 69.69 | 85.05 | 100.04 | 85.85 | 105.57 | 125.28 |
| May | 50.56 | 61.58 | 72.61 | 78.83 | 95.96 | 113.09 | 96.63 | 118.71 | 140.80 |
| June | 57.64 | 69.72 | 81.80 | 88.29 | 107.28 | 126.28 | 107.56 | 132.15 | 156.75 |
| July | 51.18 | 62.88 | 74.58 | 80.97 | 99.08 | 117.19 | 99.77 | 123.06 | 146.34 |
| August | 25.61 | 32.87 | 40.13 | 44.01 | 55.56 | 67.10 | 55.56 | 70.57 | 85.59 |
| September | 14.69 | 19.32 | 23.94 | 25.73 | 37.27 | 48.80 | 32.60 | 55.39 | 78.17 |
| October | 7.78 | 10.02 | 12.26 | 13.50 | 16.98 | 20.47 | 17.10 | 21.59 | 26.09 |
| November | 4.07 | 6.04 | 8.01 | 8.67 | 13.36 | 18.05 | 11.21 | 20.69 | 30.17 |
| December | 5.35 | 8.27 | 11.20 | 11.92 | 19.87 | 27.82 | 15.11 | 32.85 | 50.60 |

Source: Elaborated by the authors (2021).

## 4. FINAL CONSIDERATIONS

This research showed the modeling of maximum daily rainfall by EVT for the municipality of João Pessoa, Paraíba state, Brazil, from 1980 to 2019. The results show that the Gumbel distribution had the best fit for the months from January to August and October, with parameters estimated by the maximum and likelihood method. The GEV distribution proved to be more suitable for months with lower rainfall levels. Return level estimates suggest natural disasters caused by heavy rains in a single day for return periods of 2,5 , and 10 years. These results can provide subsidies for the planning of public policies to reduce heavy rain impacts.

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